

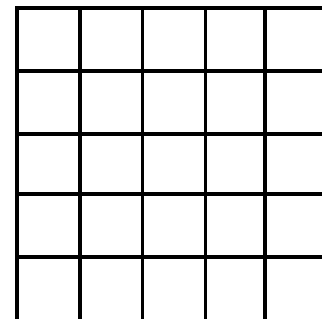
Teaching Through Problem Solving

Session 4

Prom Dress Problem

Gumball Problem

Pool →



- use curriculum expectations and mathematical processes (i.e., grades 6 to 10) to anticipate a range of solutions to a problem, to understand and describe the mathematics in student solutions, and to judge the appropriateness of problems for teaching/learning
- develop a knowledge package or landscape of clustered curriculum expectations
- use the three-part problem solving lesson design to frame the use of problems for teaching mathematics
- develop strategies and mathematics knowledge for anticipating student responses and understanding students' mathematical thinking
- experience strategies for developing students' mathematical communication through the discourse of a math-talk learning community, teacher recording strategies (blackboard writing – mathematical annotations), and coordination and recording of discussion (bansho)
- vary the structure of the problem for students to practise new learning and to provoke use of some strategies and not others, moving towards strategies that can generalize

Bansho Reinvented for Ontario Classrooms - “Ontario” Bansho

- a **mathematics instructional strategy** that provokes students' mathematical thinking to be explicit when solving problems through the organization and annotation of student work samples and classroom discourse
- a **classroom artefact** produced collectively by students and teacher that publicly displays the mathematical relationship among students' solutions; could be used as a mathematics landscape for learning or as a mathematics anchor chart
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Tt Uu Vv Ww Xx Yy Zz

to up
the under
that
they
there
then
them

was
with
we
you
yes

Tuesday March

NAME: _____
DATE: _____

10	10	10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10	10	10

There are 25 people in the class. I know 10 + 10 = 20. I know 10 + 5 = 15. I know 10 + 15 = 25.

ALLEN

PARS

COMAS

LARND

AN KNPR

6 + 9 = 15

GARDIN

CONSTANLS

I think it is 25 because I used the bells and I start at 9. So I could have some time to think and I could on I added Mrs. Allen class and I added 24 + 1 = 25 and I added Mrs. Allen class and I added 24 + 1 = 25.

615145.25

615145.25

615145.25

615145.25

7116 = 25

7116 = 25

7116 = 25

7116 = 25

Splitting numbers to make it easier.

counting on from the big number

counting on and keeping track

Counting all together by ones

NAME: _____
DATE: _____

I know 10 + 10 = 20. I know 10 + 5 = 15. I know 10 + 15 = 25.

There are 25 people in the class. I know 10 + 10 = 20. I know 10 + 5 = 15. I know 10 + 15 = 25.

16 + 9 = 25

16 + 9 = 25

16 + 9 = 25

16 + 9 = 25

16 + 9 = 25

16 + 9 = 25

16 + 9 = 25

16 + 9 = 25

16 + 9 = 25

16 + 9 = 25

16 + 9 = 25

16 + 9 = 25

Splitting 9 into 4 + 5

16 + 9 = 25

16 + 9 = 25

16 + 9 = 25

16 + 9 = 25

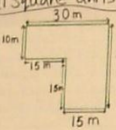
...expressions
 # Words, language
 # your imagination
 - your imagination is anything you want, your
 schema is anything you learn

Secret Valley

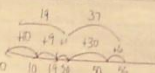
USE YOUR SCHEMATA REFLECT

We have different mental images
 because we have different schema
 for Mental Images for the poem
 Morning

Social Studies

Ms Burgess' Backyard
 Area = 61 Square Units
 110m

 Perimeter = 110m
 $30 + 25 + 10 + 15 + 15 + 15$
 $30 + 20 + 5 + 10 + 10$
 $50 + 10 + 20 + 30$
 $60 + 20 + 30$
 $80 + 30 = 110$

Our Addition Strategies

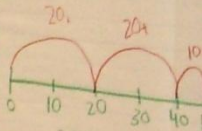
$19 + 37$ (Decomposing)
 $10 + 30$ $9 + 7$
 40 16
 $40 + 10 = 50$ $50 + 6 = 56$
 regrouping
 $10 + 40 + 6 = 56$


1 ten + 9 ones
 3 tens + 7 ones
 4 tens + 16 ones
 4 tens + 1 ten + 6 ones
 5 tens + 6 ones = 56
 or 50 + 6 = 56

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Grade 2
 ① Ms Branco wants to put up fences around the 2 Playscapes and the Kindergarten Area. She needs to know the total distance around the three play areas.
 Grade 2
 ② Ms. Branco would like to know how many bags of woodchips in total will be needed to cover the three play areas. 1 square unit represents 1 bag of wood chips.

18 + 8 = 26
 14 + 15 = 29
 23 + 23 = 46
 I DECORATED
 10 + 10 = 20
 20 + 20 = 40
 20 + 20 = 40
 20 + 20 = 40
 60 + 10 = 70

Grade 2 #2

 20 + 20 = 40
 We added the number together

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Combined Grades 2 and 3 - Perimeter and Area (Part A)

Tf Gg Hh Ii Jj Kk Ll Mm Nn Oo

Grade 2
 Ms Branco wants to put up fences around the 2 Playscopes and the Kindergarten Area. She needs to know the total distance around the three play areas.
 Grade 2
 Ms. Branco would like to know how many bags of woodchips in total will be needed to cover the three play areas. 1 square unit represents 1 bag of wood chips.

1. Decompose
 $10 + 10 = 20$
 $20 + 20 = 40$
 $40 + 20 = 60$
 $60 + 20 = 80$
 $80 + 20 = 100$

Junior
 $10 + 10 = 20$
 $20 + 20 = 40$
 $40 + 20 = 60$
 $60 + 20 = 80$
 $80 + 20 = 100$

Primary Play ground
 Decomposing
 $10 + 10 = 20$
 $20 + 20 = 40$
 $40 + 20 = 60$
 $60 + 20 = 80$
 $80 + 20 = 100$

Grade 2 #5 Pirnie, Mahood
 $10 + 10 = 20$
 $20 + 20 = 40$
 $40 + 20 = 60$
 $60 + 20 = 80$
 $80 + 20 = 100$

Decomposing
 $20 + 20 = 40$
 $40 + 20 = 60$
 $60 + 20 = 80$
 $80 + 20 = 100$

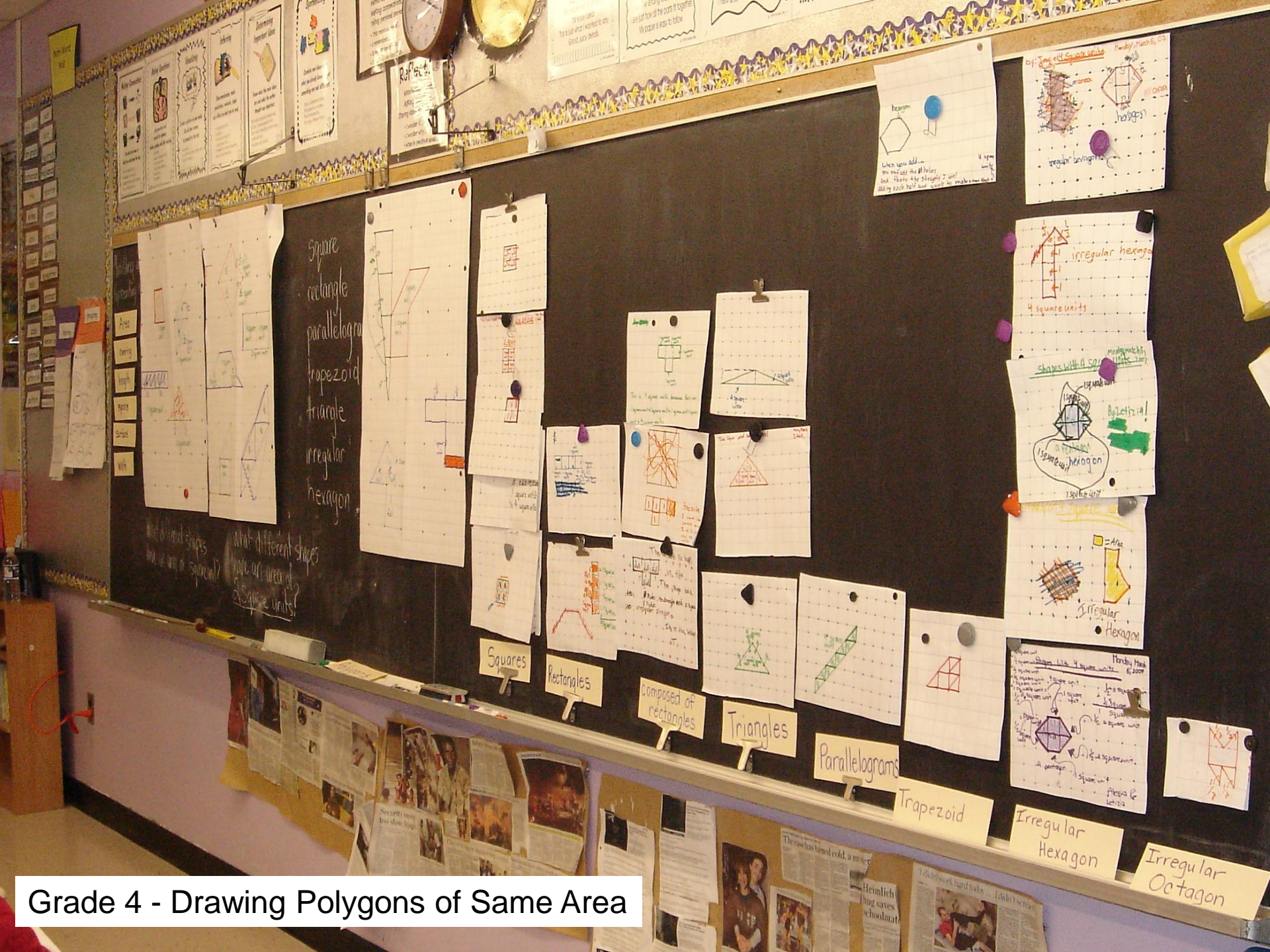
Grade 3
 ① For safety reasons, Ms. Branco would like to put a fence around our field. She needs to know the total distance around the field to know how much fencing to order.
 ② Ms. Branco wants to put new sod (grass) on our field. She needs to know how many squares of sod are needed to cover the field. 1 square of sod covers an area of 1 square unit.

One way to find perimeter around the field is to add the numbers.
 $137, 114, 32, 32, 33, 23$
 $137 + 114 + 32 + 32 + 33 + 23$
 $100 + 100 + 30 + 30 + 30 + 20 + 30 + 10 + 7 + 4 + 2 + 2 + 3 + 3$
 $200 + 74 + 60 + 22 + 50 + 33 + 40$
 $200 + 11 + 60 + 4 + 50 + 6 + 40$
 211
 $211 + 40 = 251$
 $110 + 10 = 120$
 $211 + 120 = 331$
 #2 for Grade 3's
 Numbers Words
 $30 + 30 + 30$
 $+ 30 + 30 + 30$
 $+ 30 = 210$
 $180 + 30 = 210$
 $210 + 10 = 220$
 $30 \times 7 = 210$

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Grade 4 - Drawing Polygons of Same Area



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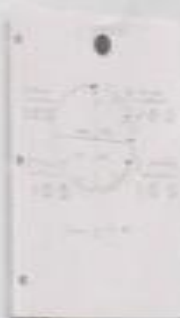
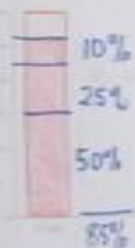
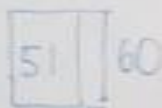
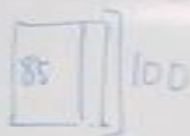
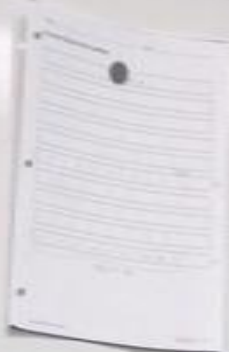
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Combined Grades 5 and 6 - Area of Irregular Figure - Using Rectangles or Using Triangles

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%, w/w
 measure
 info context



$$\begin{aligned} 85\% \text{ of } 60 &= 51 \\ 6 \times 12 &= 72 \\ 72 - 21 &= 51 \end{aligned}$$

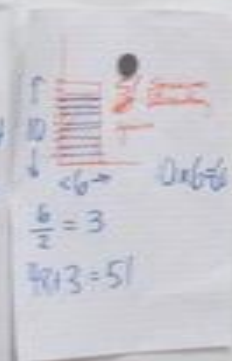
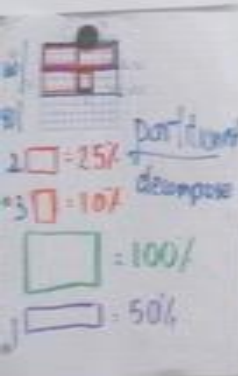
3 rows of 2
 6 rows of 2
 $2 \times 2 = 4$
 $2 \times 2 = 4$
 12 rows of 2
 24 rows of 2

85% of 60 = 51

85% of 60 = 51

85% of 60 = 51

85% of 60 = 51



$$\begin{aligned} 85\% \text{ of } 60 &= 51 \\ 6 \times 12 &= 72 \\ 72 - 21 &= 51 \end{aligned}$$

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85% of 60 = 51

85% of 60 = 51

85% of 60 = 51

85% of 60 = 51

Percent (P%)
 Anchors 50, 25, 10
 Region Array

Percent (P%)
 Anchor 10, 5%
 Region Array

Compare
 85% of 60
 and 85% of 100
 Number Line

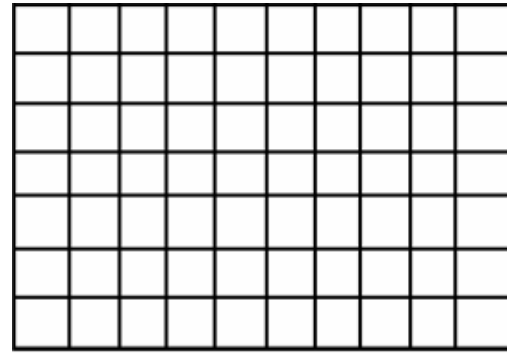
Percent Anchors
 Part to Whole
 Area - MU

Grade 7 and 8 - Representing 85% of 60 in Different Ways

Three-Part Problem Solving Lesson Design Before (Activating Knowledge)

What is similar and different about these numbers?

1, 64, 16, 4, 36, 9, 25, 49



Explain your ideas using a rectangular grid.

Our Ideas: square numbers, odd and even numbers, pattern of 1^2 , 2^2 , 3^2 , 4^2 , Arrays of 2×2 , 3×3 , 4×4 , etc., area of squares with 2 linear units by 2 linear units, 3 lu x 3 lu, 4 lu x 4 lu; use of a table to record the lengths and area on a table of values, graphing to see a quadratic relationship

What is the mathematical purpose of this activation task?

Lesson Learning Goal

- Curriculum Expectations (Math Concepts/Terms)

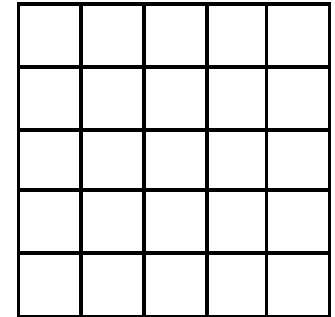
- Gr7 – model real life linear relationships graphically and algebraically
- Gr8 – model linear relationships using table of values, graphs, and equations, using a variety of tools
- Gr9 – construct tables of values, graphs, and equations, using a variety of tools to represent linear relations derived from descriptions of realistic situations
- Gr10 – construct tables of values and graphs, using a variety of tools to represent linear relations derived from the descriptions of realistic situations

Need to look at other grade specific expectations to determine the qualitative differences between these expectations ... Knowledge package, landscape, and learning trajectory

Three-Part Problem Solving Lesson Design During (Working On It) – Pool Border Problem

1a. How many one-by-one tiles are required to surround a 5x5 pool?

Pool



1b. Develop a generalization that predicts the number of tiles required to surround a square pool of any size.

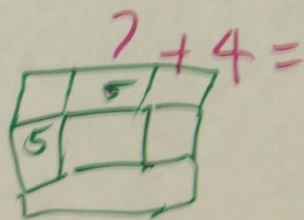
1c. Explain how your generalization relates to the size of the pool and the number of border tiles.

List grade- specific curriculum expectations that can be learned through this problem

List the math concepts, strategies, and terms that you want to make explicit?

POOL

SIDE	OUTSIDE
5	-4 ²⁴ +4
4	-4 ²⁰ +4
3	-4 ¹⁶ +4
2	-4 ¹² +4
1	-4 ⁸ +4
6	-4 ²⁸ +4
7	-4 ³² +4



1414 Table 9.46/420

Square:

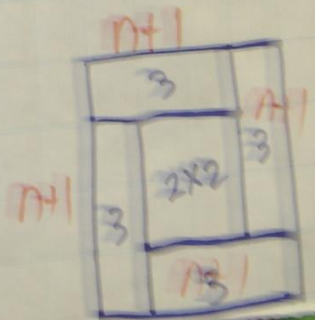
	π	Add.
1	5	$(5 \times 2) + (2 \times 1)$
2	4	$(4 \times 2) + (2 \times 2)$
3	3	$(5 \times 3) + (2 \times 3)$
5	1	$(7 \times 2) + (2 \times 5)$

Border:

$n(n+2) + 2n$

$n+2$

Side Length of Pool	Perimeter
1+1	$2 \times 4 = 8$
2+1	$3 \times 4 = 12$
3+1	$4 \times 4 = 16$
4+1	$5 \times 4 = 20$
5+1	$6 \times 4 = 24$
...	
$n+1$	$4(n+1)$



pool side length = 1

pool side length = 2

$(\text{side length} + 1) \times 4 \text{ sides} = \# \text{ of tiles}$

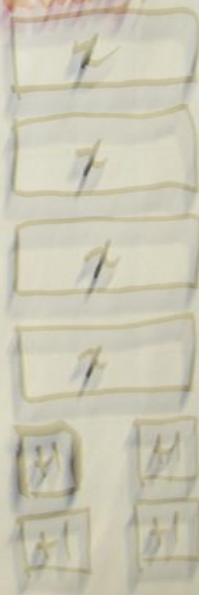
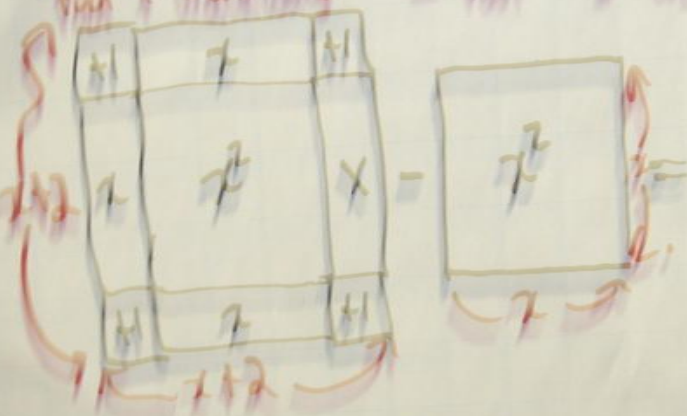
$\# \text{ of tiles} = 4(n+1)$

... etc.

$n = \text{length of pool}$

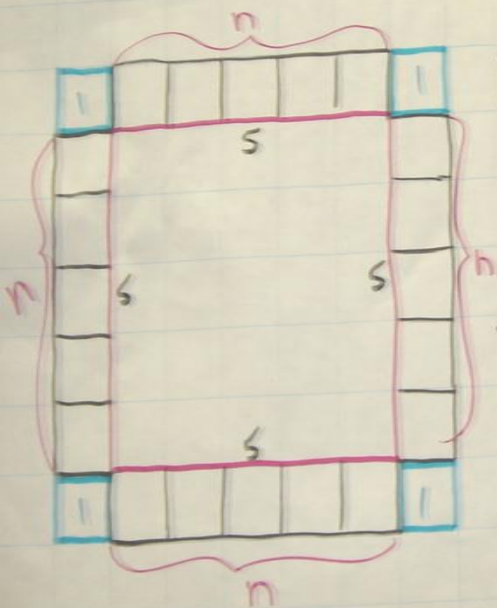
Area / Algebra Tiles Representation

Pool + walkway = Pool + walkway



$$(7+2)^2 - 7^2 = 4 \times 7 + 4$$

10 ANSWER SOLUTION



Number of tiles around
a 5x5 pool is
 $4 \times 5 + 4$

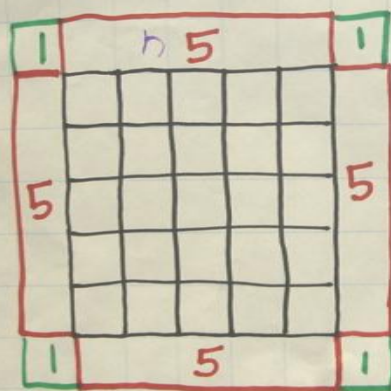
This can be generalized
to
 $4 \times n + 4$

Side length	no. of tiles
1	8
2	12
3	16
4	20
5	24
6	28
...	...
n	$4n + 4$

Consecutive terms start
at one. Number of tiles
Starts at 8,
increases by 4
each time

Rule: multiply the
side length by
four, then add
four.

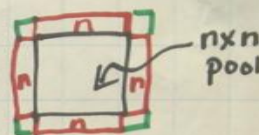
1a.



$$4 \times 5 + 4 = 24$$

$$4n + 4$$

b.



$$\text{Perimeter} = 4 \times n + 4$$

c. For an $n \times n$ pool
the number of border
tiles can be found by
multiplying the side
length by 4 (4 sides)
and adding on 4 corners

side length x y # of tiles in border

LS / RS check


$y = 4x + 4$


$24 = 4(5) + 4$

$24 = 20 + 4$

$24 = 24$

$\therefore LS = RS$

1 8  1x1

2 12  2x2

3 16

4 20

5 24

$y = 4x + b$

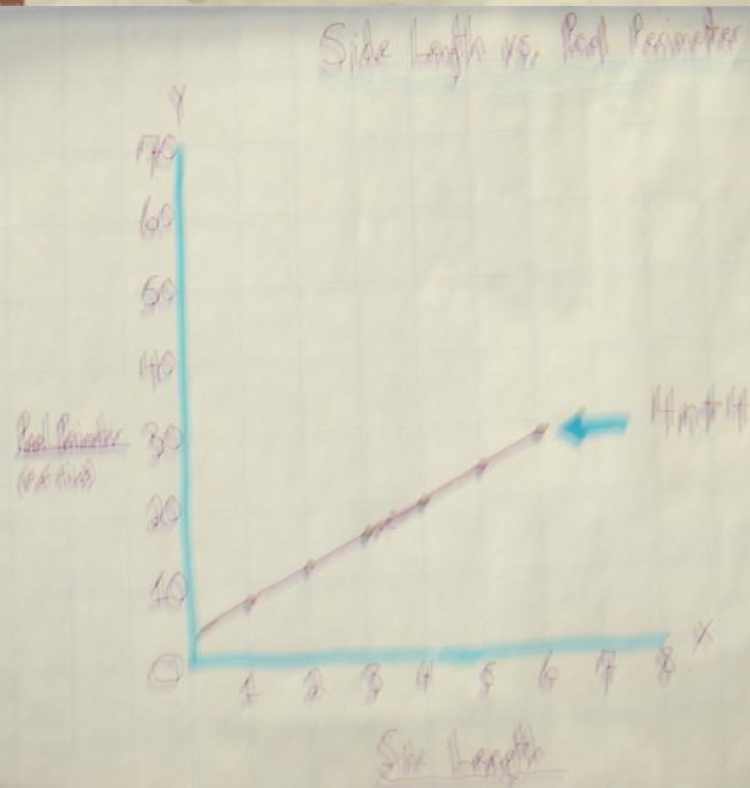
$8 = 4(1) + b$

$8 - 4 = b$

$4 = b \therefore y = 4x + 4$

of tiles in border w/o corners OR # of sides in square

CORNER TILES



side length x y # of tiles

consecutive terms start at one. Number of tiles starts at 8, increases by 4 each time

Rule: multiply the side length by four, then add four.

1 8

2 12

3 16

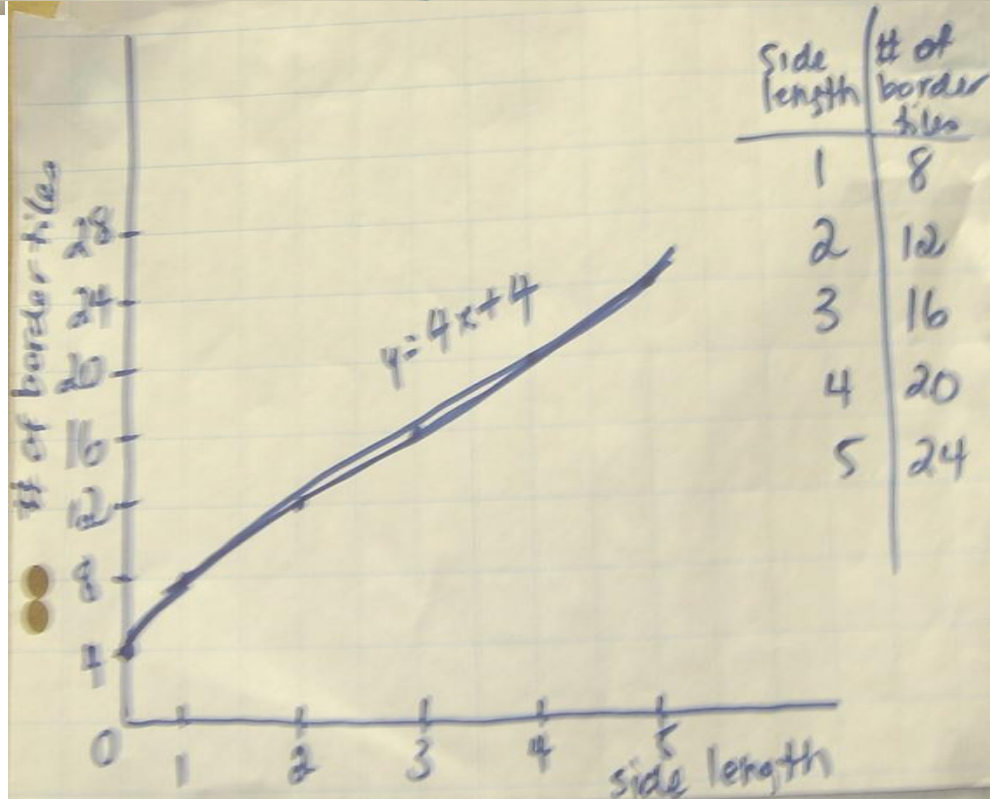
4 20

5 24

6 28

...


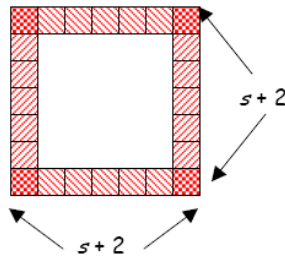
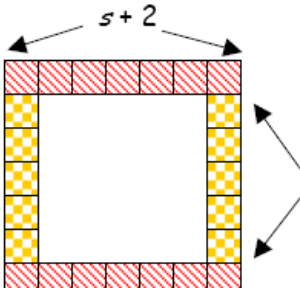
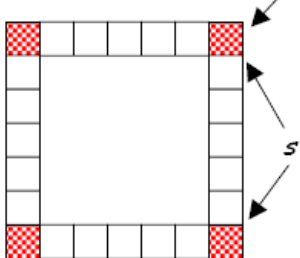
n $4n + 4$



[illegible]

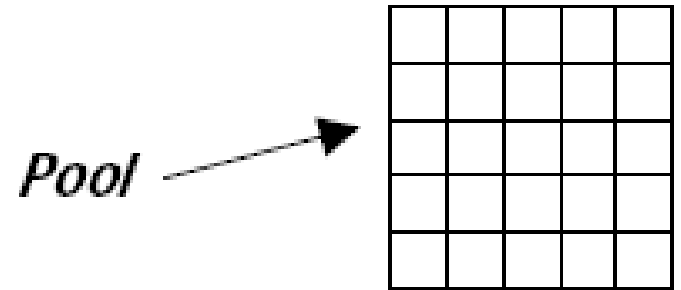
Solutions are organized by mathematical strategy (composition of parts of the pool Border, showing different algebraic expressions (akin to simplifying expressions to the Same expression $4(n+4)$); Highlights focus on the use of the data organized in a table of values (i.e., numeric cases), to build an expression showing constant and variable (in this case, the “quasi-variable”).

- What mathematics criteria will you use to organize the student solutions?
- How does the mathematics sorting criteria relate to the lesson learning goal?

Lesson/Unit of Study Title	Grade	Assessment FOR LEARNING OBSERVATION & INTERVIEW	Date																				
Mathematics Lesson Task/Problem		Learning Goal/Curriculum Expectations																					
<p>Develop a generalization that predicts the number of tiles required to surround a square pool of any size.</p> 																							
Students	Meth Thinking																						
Marc	1, 3																						
Jan	2																						
Ravi	1																						
Pat	3																						
																							
x	y or # of tiles	<table border="1" style="width: 100%; border-collapse: collapse;"> <tbody> <tr> <td>0</td> <td>4</td> <td rowspan="5" style="vertical-align: middle; font-size: 2em;">}</td> <td>-4</td> </tr> <tr> <td>1</td> <td>8</td> <td>+4</td> </tr> <tr> <td>2</td> <td>12</td> <td>+4</td> </tr> <tr> <td>3</td> <td>16</td> <td>+4</td> </tr> <tr> <td>4</td> <td>20</td> <td>+4</td> </tr> <tr> <td>5</td> <td>24</td> <td></td> <td></td> </tr> </tbody> </table>		0	4	}	-4	1	8	+4	2	12	+4	3	16	+4	4	20	+4	5	24		
0	4	}	-4																				
1	8		+4																				
2	12		+4																				
3	16		+4																				
4	20		+4																				
5	24																						
Questions About Students' Mathematics Learning		Students' Mathematical Errors																					

Three-Part Problem Solving Lesson Design After (Practice) –

- What problem(s) should we provide to enable students to practise (in pairs, individually) what they have learned?
- What learning goals (curriculum expectations) does your Practice Problem?



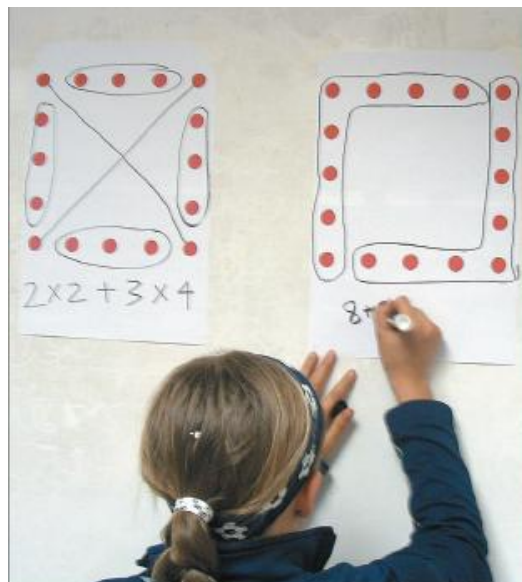
1a. How many one-by-one tiles are required to surround a 5x5 pool?

1b. Develop a generalization that predicts the number of tiles required to surround a square pool of any size.

1c. Explain how your generalization relates to the size of the pool and the number of border tiles.

Three-Part Problem Solving Lesson Design After (Practise)

- What problem(s) should we provide to enable students to practise (in pairs, individually) what they have learned?
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BLAKE E. PETERSON

Counting Dots and Measuring Area:

Rich Problems from Japan

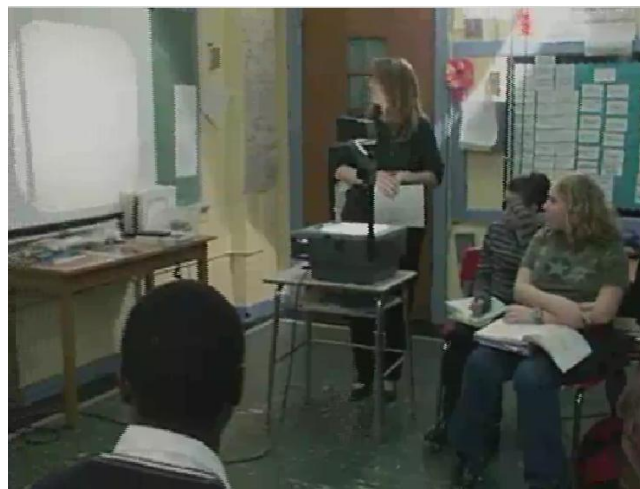
IN FALL 2003, I HAD THE OPPORTUNITY TO conduct some research on the student-teaching process in Japan. During my seven weeks of research at the junior high school affiliated with Ehime University in Matsuyama, Japan, I observed mathematics lessons taught by student teachers as well as many more lessons taught by experienced teachers. The basis for most of these lessons was wonderfully rich mathematics problems. In these lessons, a problem was posed to the students, time was given for them to explore the problem, and then solutions were discussed. Similar problem-based lessons can be found in *The Teaching Gap* (Stigler and Hiebert 1999) and *The Open-Ended Approach: A New Proposal for Teaching Mathematics* (Becker and Shimada 1997).

Some assets of these problems were the connections students were able to make and the variety of representations they were able to employ in solving

richness of these problems had a great deal to do with the connections and representations that were such a prominent part of these Japanese lessons.

Many teachers in the United States are making efforts to incorporate the Process Standards from *Principles and Standards for School Mathematics* (NCTM 2000) into teaching and learning mathematics in their classrooms. The Problem Solving Standard for Grades 6–8 states, “Problem solving is central to inquiry and application and should be interwoven throughout the mathematics curriculum to provide a context for learning and applying mathematical ideas” (NCTM 2000, p. 256). The Connections Standard for Grades 6–8 states, “If curriculum and instruction focus on mathematics as a discipline of connected ideas, students learn to expect mathematical ideas to be related. Rich mathematical tasks prompt students to use and develop mathematical understandings and connections” (NCTM 2000, p. 275). If students

Other Lesson Approaches to the Pool Border Problem ... What Do You Think?



Criteria for Lesson Analysis for Teaching Through Problem Solving

- Before? During? After?
- Math Content coherence?
- Making Mathematical Connections?
- Who does the Mathematics Work?
- Nature of Mathematics Learning?
- Nature of the Mathematical Work?
- Kind of Mathematics Work Expected?
- Content Elaboration by Whom?

Developing a Math Talk Community

- Anticipating Student Responses to Improve ...



Anticipating Student Responses to Improve Problem Solving

ANN H. WALLACE

Read 1 page ...
Key idea or strategy that supports the development of a math-talk community.

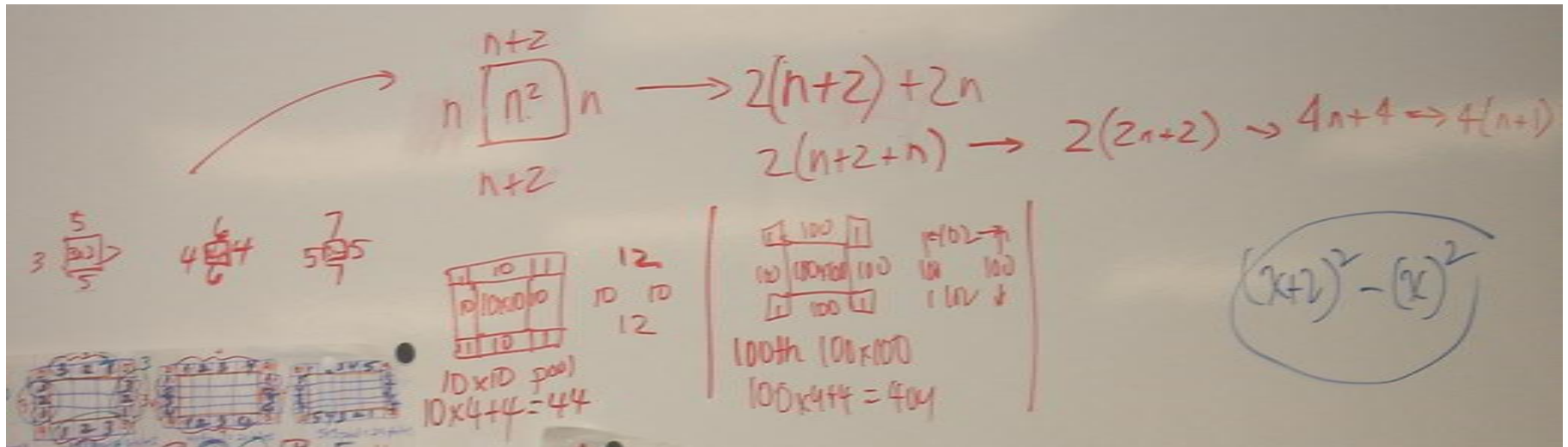
I HAVE BEEN A MATHEMATICS TEACHER AND EDUCATOR THROUGHOUT ALL FOUR publications of the *Standards* documents (NCTM 1991, 1995, 1989, and 2000). Over the years, while concentrating on improving various aspects of my teaching, specifically, improving my students' ability to problem solve, I have been perplexed to see students pick numbers out of a problem and perform an operation with no regard for the context. To address this issue by teaching problem-solving lessons made me realize that I did not know the difference between students solving a problem and

Linking to Differentiating Instruction

- Teach to the group but differentiate consolidation – open-routed problems provoke differentiated responses
- Teach different things to different groups – they listen in relation to their mathematical readiness, curiosity, and confidence
- Provide individual learning packages as much as possible - Consolidation
- Personalize both instruction and assessment – who is included and not included; what evidence of learning have you gathered from the classroom collective and individually
- Key concepts, choice, prior assessment

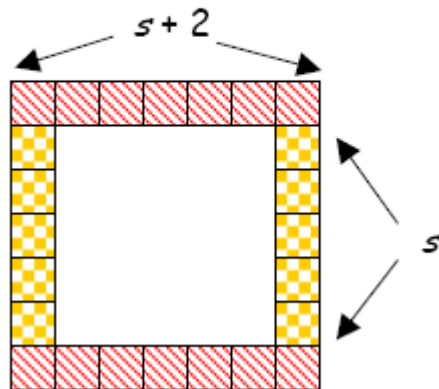
Three-Part Problem Solving Lesson Design

- Before (Comparing 2 Solutions)



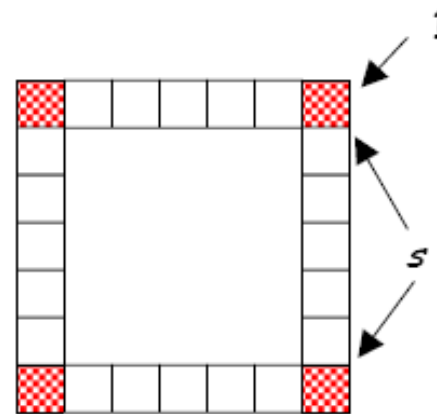
2. Top/bottom and 2 sides:

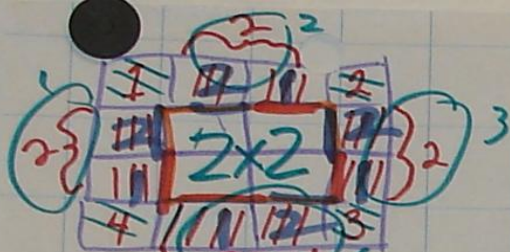
$$n = 2(s+2) + 2s$$



1. Sides + corners:

$$n = 4s + 4$$





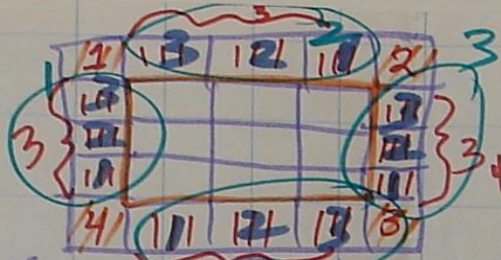
of tiles = 4 sides + 4 corners

$$2 \times 4 + 4 = 12$$

$$2 \times 2 \text{ pool} = 12 \text{ tiles}$$

$$2 + 2 + 2 + 2 + 4 = 12$$

$$4 \times 3 = 12$$



3x3 pool = 16 tiles

$$3 \times 3 + 4$$



4x4 pool = 20 tiles

$$4 \times 4 + 4$$



5x5 pool = 24 tiles

$$5 \times 5 + 4$$

Multiplicand

Multiplier

Constant

$X = W = L$

COUNTED

of square tiles around pool

Decompose

Expression

$4x + 4 = y$

# of square tiles	Multiplicand	Multiplier	Variable
12	2	2	2
16	3	3	3
20	4	4	4
24	5	5	5
	N	N	N

1 tile

$$4 + 2 + 2 + 4 + 4$$

$$4 + 4 + 4 + 4 + 4$$

$$4 + 4 + 4 + 4 + 4 + 4$$

$$4 + 4 + 4 + 4 + 4 + 4 + 4$$

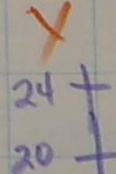
$$4(2) + 4 = 12$$

$$4(3) + 4 = 16$$

$$4(4) + 4 = 20$$

$$4(5) + 4 = 24$$

$$4N + 4$$



$$y = 4x + 4$$

neg. # of squares \rightarrow not possible \rightarrow X-intercept = (, 0)

Y-intercept = (0,)
(not possible, 4 corners)

Multiplicand
(size of group)
Multiplier
(# groups)
Variable

Three-Part Problem Solving Lesson Design

- During – Prom Dress and Gum Problem

Veronica and Caroline found the perfect prom dress which cost \$80, but neither had enough money to buy it at the time.

Veronica put \$20 aside that night and has been putting aside an additional \$5 a day, since then.

Caroline put aside \$8 every day since the day after she saw the dress. .

Heather says, “Wow! Caroline has more money saved.”

How many days has it been since Veronica and Caroline began saving?

Ken and his brother enjoy chewing gum. One day, the boys go to the candy store and buy several packages of gum.

Ken bought 18 ten-piece packages of gum, and his brother bought five-piece packages of gum.

Every day, each of the boys finishes one whole pack of gum. One day, they looked at how much gum each boy had. Ken noticed that his brother had more pieces of gum than he had.

How many days has it been since the boys bought the gum?

Three-Part Problem Solving Lesson Design - After (Consolidation)

- Show the curriculum expectations are the learning goal of your problem?
- Show the mathematics terms, symbols, key concepts, strategies are you making explicit to support the learning goal?
- Show the mathematics sorting criteria that you are using to show a recursive elaboration (transformation) across the solutions?
- Show the “Highlights” chart (matome) to make explicit how the different aspects of the student solutions contribute to the key mathematics that you want to be explicit .

Three-Part Problem Solving Lesson Design - **After (Practice)**

- What problem(s) should we provide to enable students to practise (in pairs, individually) what they have learned?
- What learning goals (curriculum expectations) does your Practice Problem?

Looking Back - Margaret Smith

- How are the problems similar? Different?
- How are the lesson designs similar? Different?

Beyond Presenting Good Problems How a Japanese Teacher Implements a Mathematics Task

Margaret Smith

FOR several years teachers have been encouraged to use tasks that engage students in mathematical thought processes. For example, the recommendations by the National Council of Teachers of Mathematics (NCTM) in both the *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989) and the *Principles and Standards for School Mathematics* (NCTM 2000) encourage teachers and curriculum developers to use problems that go beyond practicing routine procedures to problems that help students build mathematical connections and develop and apply mathematical concepts. The expectation is that, by incorporating problems that engage students in mathematical thought processes, teachers will provide students with opportunities to conjecture, reason, and develop new mathematical ideas. Engaging students in such discussions is relatively new for both teachers and students. To help teachers learn some useful strategies, this article offers examples and discussions of ways teachers can implement problems to help students engage in mathematical thought processes.

TASKS THAT HELP STUDENTS MAKE CONNECTIONS

The initial release of data from the Third International Mathematics and Science Video Study (TIMSS Video Study) promoted a lot of discussion

The data used in this article were generated as part of requirements for a doctoral dissertation at the University of Delaware using video data from the TIMSS Laboratory at the University of California, Los Angeles. I wish to thank James Hiebert for his wisdom and guidance in the project, as well as

“Ontario” Bansho

- as an Instructional Strategy

To provoke students' mathematical thinking to be explicit when solving problems through the organization and annotation of student work samples and classroom discourse

Preparation – Teacher:

- cleared black board space
- chose lesson problem based on grade-specific expectations
- solve the problem prior to the lesson to anticipate a range of solutions, to decide on the math tools students need, and to anticipate the key mathematical aspects of solutions
- prepare (if possible) mathematical labels for categories of the solutions
- provide square grid chart paper (in eighths), markers so that solutions are visible to students

Actions – Teacher:

- activate student knowledge and experience at start of lesson
- circulate among the students as they are solving a problem to discern the range of the solutions and the mathematical connections to prompt
- sort and classify their solutions (like a concrete graph) according to mathematical criteria related to the learning goal(s) of lesson
- select which students will share their solutions 1st, 2nd, 3rd, so to build mathematics knowledge through analysis and discussion

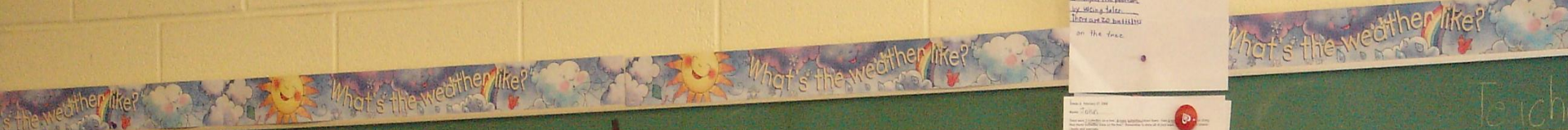
“Ontario” Bansho

- As a Classroom Artefact

An artefact produced collectively by students and teacher that publicly displays the mathematical relationship among students' solutions; could be used as a mathematics landscape for learning or as a mathematics anchor chart.

- The flow of three-part lesson (e.g., problems, key words, strategies) are posted on the blackboard, from left to right
- Student solutions to lesson problem are categorized and labelled like a concrete graph (from 4 solutions to all solutions)
- Student solutions are mathematically annotated to highlight key concepts, representations, strategies, vocabulary, and symbols
- Student work samples and ideas from previous lessons are posted to revisit prior mathematical ideas and strategies

something
 without
 football
 doctor
 library
 teacher
 island
 puppet
 umbrella
 dogs
 cats
 wishes
 foxes
 families
 Can you think of more plural words?
 All numbers and colours are adjectives



Strategies
 • doubles plus 1
 $8 + 9 = 17$
 $10 + 1 = 11$
 $17 + 1 = 18$
 • Nearly Neighbours
 $6 + 8 = 14$
 $7 + 7 = 14$
 • Making 10
 $9 + 8 = 17$
 $10 + 1 = 11$
 $11 + 6 = 17$
Manipulatives
 • cubes/counters
 • beads
 • ten frames/20 frames

Math Problem
 There were 7 butterflies on a tree.
 8 more butterflies joined them. Then
 5 more butterflies came along. How
 many butterflies were on the tree?
 Remember to show all of your work and explain your
 answer clearly and precisely.
 • tallies -
 • number sentence

There were 7 butterflies on a tree.
 8 more butterflies joined them. Then
 5 more butterflies came along. How
 many butterflies were on the tree?
 I used tallies to solve the problem.

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 many butterflies were on the tree?
 I used tallies to solve the problem.

September
 7
 8
 5
 The answer is 20.
 I used the number line to solve the problem.
 There are 20 butterflies on the tree.

I used the number line to solve the problem.
 There are 20 butterflies on the tree.
 8 + 5 + 5 = 20
 5 + 5 + 5 = 20

There are 20 butterflies on the tree.
 8 + 5 = 13
 13 + 5 = 18
 18 + 2 = 20

I used tallies to solve the problem.
 There are 20 butterflies on the tree.
 8 + 5 + 5 = 20

I used tallies to solve the problem.
 There are 20 butterflies on the tree.
 8 + 5 + 5 = 20



Teaching
 Mrs. Coffey
 Ms. Kubacki
 Mrs. Nazzari

I used tallies to solve the problem.
 There are 20 butterflies on the tree.
 8 + 5 + 5 = 20

I used tallies to solve the problem.
 There are 20 butterflies on the tree.
 8 + 5 + 5 = 20

I used tallies to solve the problem.
 There are 20 butterflies on the tree.
 8 + 5 + 5 = 20

I used tallies to solve the problem.
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Counting by 1's

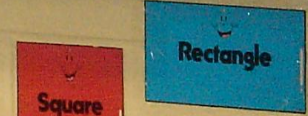
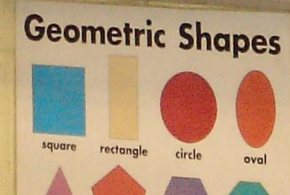
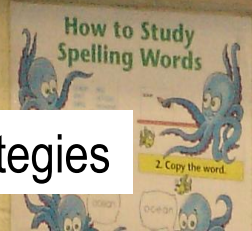
Counting Up (larger # first)

Making 10

Groups of 5

Trading/Exchanging #'s

Doubles



Grade 2 - Addition Strategies

- To discern the range and the mathematical relationships between ideas, strategies, and models of representation used to solve a problem by students.

[illegible]

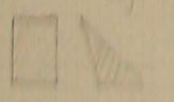
HOMEWORK

Math Communication On Post

- draw picture from problem
- linear dimensions (cm, mm)
- use of formulas
- draw different shapes/
- faces to explain thinking
- the use of nets
- be specific
- P, H, W
- ✓ ✓ ✓

Area

- length x width (formula)
- square units (unit)
- base x height (formula)
- base x height (formula Δ)
- tools \rightarrow ruler \rightarrow length, measure

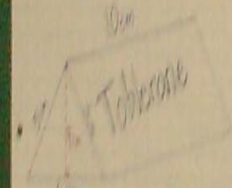


Tobacco Package Problem

Could this cigarette bar be covered using less than 150cm² of cardboard?

Triangle prism face

- 2 triangles
- 3 rectangles




Tobacco Problem


Q1. How much cardboard is needed to make the package?

Q2. What is the area of the package?

Q3. What is the perimeter of the package?



Triangle



Q1. How much cardboard is needed to make the package?

Q2. What is the area of the package?

Q3. What is the perimeter of the package?

Triangle



Q1. How much cardboard is needed to make the package?

Q2. What is the area of the package?

Q3. What is the perimeter of the package?

Strategies to use:

- area of rectangular face
- area of triangular face
- How much cardboard (more or less than 150cm²)

Triangle




Q1. How much cardboard is needed to make the package?

Q2. What is the area of the package?

Q3. What is the perimeter of the package?

Triangle




Q1. How much cardboard is needed to make the package?

Q2. What is the area of the package?

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Triangle

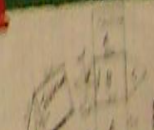


Q1. How much cardboard is needed to make the package?

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Q3. What is the perimeter of the package?

Triangle



Q1. How much cardboard is needed to make the package?

Q2. What is the area of the package?

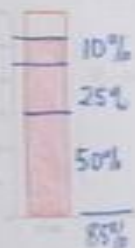
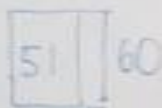
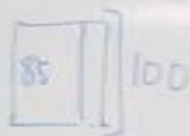
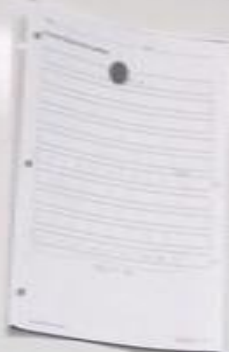
Q3. What is the perimeter of the package?

“Ontario” Bansho as a Mathematics Professional Learning Strategy

It develops teacher's knowledge of mathematics *for teaching*, to construct a mathematics landscape evoked through the solving of problems

- In preparation for teaching mathematics, solve a mathematics problem in different ways with colleagues
- Organize the solutions to show the mathematical relationship between the solutions, often in a mathematical developmental continuum
- Use this artefact to construct a mathematics landscape (often across strands) to explain the range of mathematical ideas, strategies, and models used to solve a problem

%, w/w
 measure
 info context



$$\begin{aligned} 85\% \text{ of } 60 &= 51 \\ 6 \times 12 &= 72 \\ 72 - 21 &= 51 \end{aligned}$$

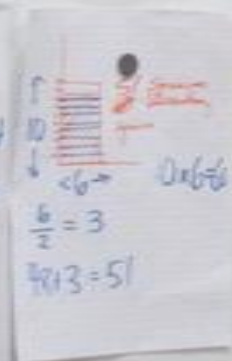
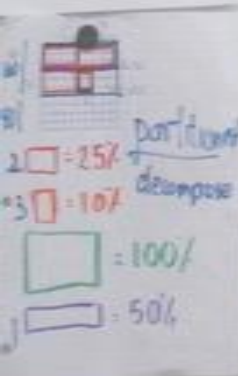
3 rows of 2
 6 rows of 2
 $2 \times 2 = 4$
 $2 \times 2 = 4$
 12 rows of 2
 24

85% of 60 = 51

85% of 60 = 51

85% of 60 = 51

85% of 60 = 51



$$\begin{aligned} 85\% \text{ of } 60 &= 51 \\ 6 \times 12 &= 72 \\ 72 - 21 &= 51 \end{aligned}$$

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Percent (P=) Anchor 50, 25, 10
 Region Array

Percent (P=) Anchor 10, 5%
 Region Array

Compare 85% of 60 and 85% of 100
 Number Line

Percent Anchor (Part to Whole)
 Area - M.U.

Grade 7 and 8 - Representing 85% of 60 in Different Ways

Learn Mathematics *for* Teaching

(Ball, 2005)



- Figure out why procedures work, not just how to do them
- Try to solve problems in more than one way
- Listen to and probe others' thinking, especially when struggling
- Study students' thinking and work
- Talk in class; practise speaking mathematics

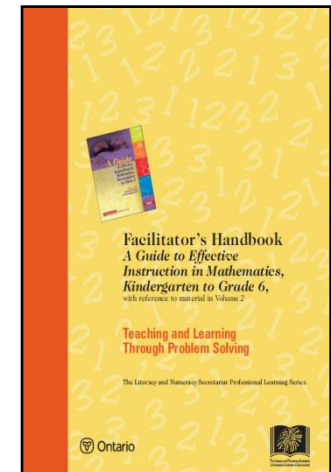
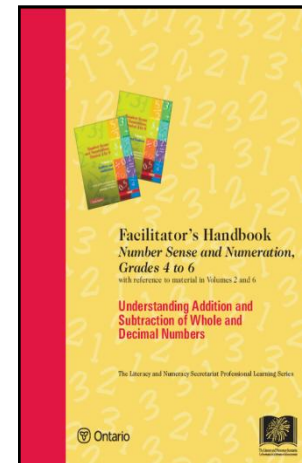
LNS Resources Highlighting Bansho

LNS Webcasts – www.curriculum.org

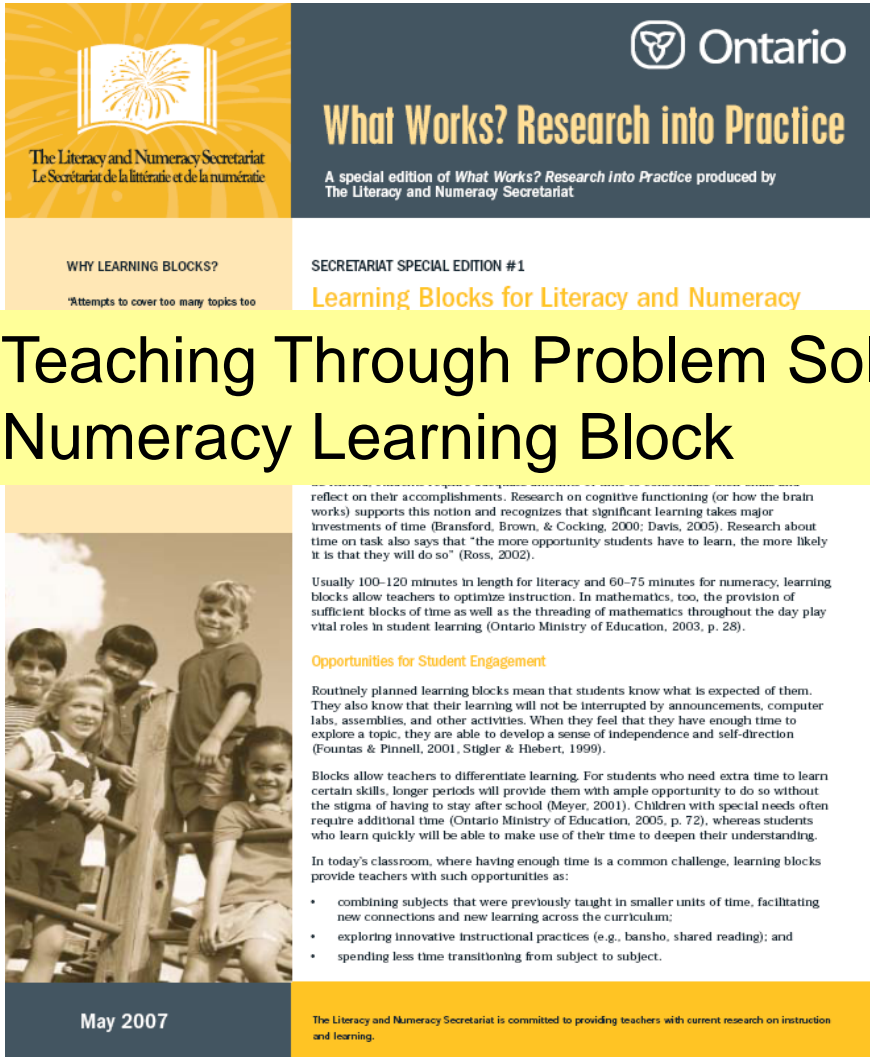
- March 2007 - Making Mathematics Accessible for all Students (Grades 1, 2/3, 4, 5/6)
- June 2007 - Coaching for Student Success (Gr3)
- March 2008 - Investigating High Yield Strategies (Gr6)

LNS Facilitator Handbooks at www.curriculum.org (Coaching Institute – Numeracy Resources)

- Addition and Subtraction
- Multiplication and Division
- Teaching Through Problem Solving



Other LNS Resources



What Works? Research into Practice
A special edition of *What Works? Research into Practice* produced by The Literacy and Numeracy Secretariat

SECRETARIAT SPECIAL EDITION #1
Learning Blocks for Literacy and Numeracy

WHY LEARNING BLOCKS?
*Attempts to cover too many topics too

reflect on their accomplishments. Research on cognitive functioning (or how the brain works) supports this notion and recognizes that significant learning takes major investments of time (Bransford, Brown, & Cocking, 2000; Davis, 2005). Research about time on task also says that "the more opportunity students have to learn, the more likely it is that they will do so" (Ross, 2002).

Usually 100-120 minutes in length for literacy and 60-75 minutes for numeracy, learning blocks allow teachers to optimize instruction. In mathematics, too, the provision of sufficient blocks of time as well as the threading of mathematics throughout the day play vital roles in student learning (Ontario Ministry of Education, 2003, p. 28).

Opportunities for Student Engagement

Routinely planned learning blocks mean that students know what is expected of them. They also know that their learning will not be interrupted by announcements, computer labs, assemblies, and other activities. When they feel that they have enough time to explore a topic, they are able to develop a sense of independence and self-direction (Fountas & Pinnell, 2001; Stigler & Hiebert, 1999).

Blocks allow teachers to differentiate learning. For students who need extra time to learn certain skills, longer periods will provide them with ample opportunity to do so without the stigma of having to stay after school (Meyer, 2001). Children with special needs often require additional time (Ontario Ministry of Education, 2005, p. 72), whereas students who learn quickly will be able to make use of their time to deepen their understanding.

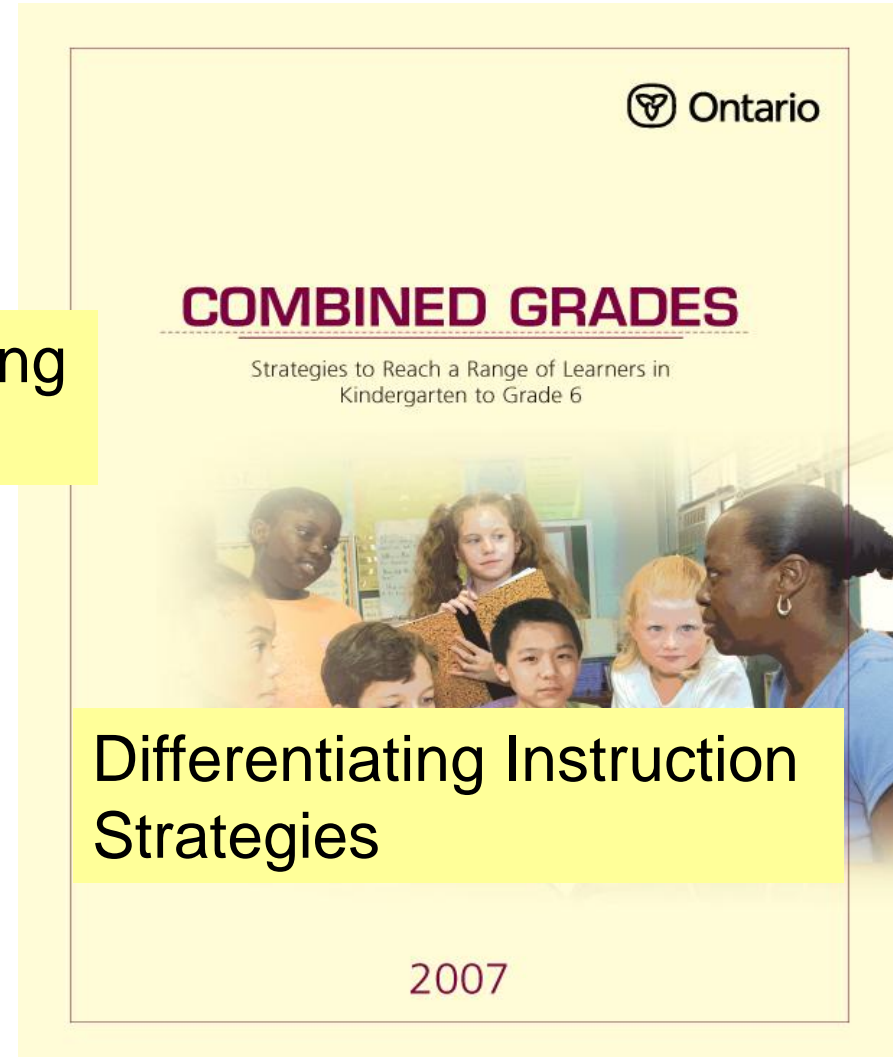
In today's classroom, where having enough time is a common challenge, learning blocks provide teachers with such opportunities as:

- combining subjects that were previously taught in smaller units of time, facilitating new connections and new learning across the curriculum;
- exploring innovative instructional practices (e.g., bansho, shared reading); and
- spending less time transitioning from subject to subject.

May 2007

The Literacy and Numeracy Secretariat is committed to providing teachers with current research on instruction and learning.

<http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/capacityBuilding.html>



COMBINED GRADES
Strategies to Reach a Range of Learners in Kindergarten to Grade 6

Differentiating Instruction Strategies

2007

<http://www.edu.gov.on.ca/eng/literacynumeracy/combined.html>